Distributed Spectral Efficiency Optimization At Hotspots Through Self Organisation of BS Tilts

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***Abstract*— Pop up traffic hotspots i.e. geographically concentrated user pockets are a time persistent reason behind poor user experience in wireless cellular system. Spatio temporal unpredictability of such pop up hotspots renders them difficult to be designed out in the planning phase of the cellular system hence dynamic and adaptive solutions are required to cope with them in an impromptu manner. In this paper we present a novel solution that addresses this problem by dynamically enhancing spectral efficiency in hotspot regions through optimisation of system wide BS antenna tilts in distributed fashion. Unlike most of the existing works that provide solutions for hotspot relief, our solution does not rely on load transferring to neighbour cells; rather it dynamically enhances the overall spectral efficiency and thus capacity of the system by jointly optimising antenna tilts of multiple adjacent cells with respect to hotspot locations in those cells. Furthermore, being designed on the principles of self organization in biological system, our solution is self organising and can improve the user spectral efficiency in a system by upto 1b/s/Hz in presence of hotspots with no significant overheads.**

I. INTRODUCTION

User distribution in real world is not uniform as often considered during Wireless Cellular System (WCS) planning and design phase as well as in simulation based evaluations. In real world, high user densities intermittently occur near points of attractions e.g. theaters, shopping malls, event venues, beaches and office complexes etc. These concentrated pockets of user manifest themselves as traffic *hotspots* that pop up at random times and at random locations in the coverage area. These hotspots become a source of low user satisfaction because of the congestion they might cause. Furthermore, pop up hotspots also can have adverse effect on overall performance of the system when they occur in the in areas where interference or shadowing is high. In such scenario only low order modulation and coding schemes can be used by the system to provide service to the users in these hotspots. Therefore, such hotspots can cause a significant proportion of user population to be served with low spectral efficiency. This in turn deteriorates the QoS and system wide throughput in WCS.

The pop up hotspots, in general, are hard to be designed for, during the WCS deployment phase because of their unpredictability both in time and space. An over safe design to accommodate potential pop up hotspots is not feasible option either, as it can be expensive and may result in under utilization of resources itself. To this end, in this paper we provide a novel Self Organising (SO) solution that resolves hot hotspot problem by dynamically maximising the user link spectral efficiency for users in hotspot. The basic idea is to enhance the average user Signal to Interference ratio (SIR) by focusing better antenna gains dynamically at locations of pop up hotspots, through optimization of *system wide antenna tilts* but in distributed and self organising manner. Numerical results show that in addition to improving spectral efficiency of hotspot user, this solution can can also improve the overall spectral efficiency of the system in face of hotspots.

There have been several studies on tilt optimization [1][1] most of which focused on improving the coverage and capacity in general for GSM [2], CDMA [3]–[5], HSDPA [6] and LTE [7]–[11] based WCS. The approaches taken in all these works can be divided in two main categories.

In first category fall the schemes that use antenna tilt basically as an *interference* reduction tool. Such schemes adapt antenna tilt to minimise interference and thus improve overall throughput of the system. These type of schemes are mostly investigated in context of CDMA based WCS to control inter cell interference [2], [4] with no consideration for hotspots.

Second category consist of schemes that exploit antenna tilt as a tool to control effective *coverage* and thus control the load in the cell to achieve *load balancing* either in the CDMA based WCS [3], [5], [6] or OFDMA based WCS [8]. Out of these the most relevant to the scope of this paper are works presented in [3] and [5]. Both [3] and [5] deal with hotspot through antenna tilting but for CDMA and HSDPA respectively unlike our work where we address this problem for OFDMA based WCS. The most important difference between the two systems in this context lies in the fact that unlike the soft handover in CDMA based WCS, in OFDMA based WCS the handover is hard handover i.e. this may involve a change of carrier frequency. This change of carrier frequency increases the complexity and overheads associated with handovers compared to the soft handovers in CDMA based WCS. Since pop up hotspots are acutely dynamic in both time and space, therefore, the compensating mechanisms for them, that require excessive handover among cells do not remain a attractive solution for OFDMA based WCS. The tilting mechanism proposed in both [3] and [5] are based on the basic idea of tilting down the overloaded cell antenna to reduce its effective coverage area in order to shift its load to neighbouring cells. Such tilting mechanisms are shown to rely on excessive handovers

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[1] A detailed survey on tilt optimization can be found in our work in [1].

from overloaded cell to adjacent cells [3]. As discussed above, such handovers can be manageable softly in CDMA, but in OFDMA based networks like LTE and LTE-A they are hard handovers and undermine the practicality of this approach.

To the best of our knowledge, the tilt optimisation based solution for hotspot based user distribution presented in this paper is novel and essentially different from the two approaches taken for tilt optimisation explained above and generally used in literature. Although we use antenna tilts to deal with hotspot , but unlike both of the works [3] and [5] , our work does not seek hotspot relief through load balancing achieved by antenna down tilting of over loaded sectors and thereby triggering handovers from that cells to neighbour cells. Neither we present a scheme to tilt down antennas for interference minimization in the system in general. Rather we introduce a novel concept of traffic's Center of Gravity (CG) to represent hotspot based user geographical distribution in a cell. By building on this concept we then develop a unique adaptive and yet scalable and agile mechanism where a pre-determined set of neighbouring cells jointly optimize their tilts to focus their antenna gains at the CG's e.g hot spots in those cells. This antenna tilt optimization is performed in distributed manner but on system wide scale by using another novel concept of *triplet* of most interdependent sectors. Thus, this mechanism can improve the overall spectral efficiency of WCS in face of a realistic non homogenous spatio temporarily varying user distribution. Since, unlike the hotspot relief approaches used in [3], [5] our approach does not achieve hotspot relief by transferring load to nearby under loaded cells, rather it relieves congestion by enhancing the spectral efficiency at hotspots, therefore, it does not necessitate handovers. The SO nature of this solution enabled by distributed nature and low complexity; and the fact that solution can be implemented through electronic antenna tilting based on local feed back only, makes it pragmatic solution for pop up hotspots.

The rest of this paper is organised as follows. In section II we present system model, assumptions and problem formulation. In order to achieve a SO solution, in section III we propose a way to decompose the system wide problem into local subproblems as inspired by SO systems in nature. Solution methodology for local subproblems is also presented in this section. Section IV presents numerical results and section IV concludes this paper with remarks on pragmatic implementation aspects and future work directions.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a multicellular network with each base station having three sectors as shown in figure 1. Let $\mathcal{N}$ denote the set of points corresponding to the transmission antenna location of all sectors and $\mathcal{K}$ denote the set of points representing the locations of users in the system. The geometric Signal to Interference Ratio i.e. SIR perceived on the downlink at a location k being served by n^{th} sector can be given as:

$$\gamma_k^n = \frac{P^n G_k^n \alpha \left(d_k^n\right)^{-\beta}}{\sum_{\forall m \in \mathcal{N} \setminus n} \left(P^m G_k^m \alpha \left(d_k^m\right)^{-\beta}\right)} \; m, n \in \mathcal{N}, k \in \mathcal{K} \quad (1)$$

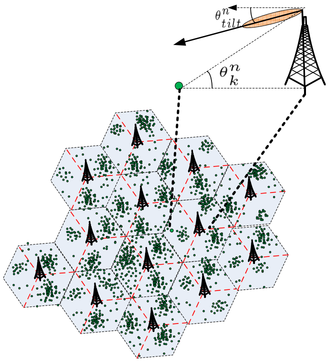

Fig. 1. System model for problem formulation. Red circles show location of hot spots in some sectors while others have uniform user distribution.

where P is transmission power, d is distance between transmmitng sector and receiving user location. α and β are pathloss model coefficient and exponents respectively. G is antenna gain and for 3GPP and LTE and LTE-A, it can be modelled as in [8] and written in dB as:

$$G_k^n = \lambda_v \left(G_{max} - \min\left(12\left(\frac{\theta_k^n - \theta_{tilt}^n}{B_v}\right)^2, A_{max}\right)\right) + \lambda_h \left(G_{max} - \min\left(12\left(\frac{\phi_k^n - \phi_a^n}{B_h}\right)^2, A_{max}\right)\right) \quad (2)$$

where θ_k^n is the vertical angle in degrees from k^{th} location of user to n^{th} sector and θ_{tilt}^n is the tilt angle of the n^{th} sector as shown in figure 1. The ϕ_k^n is horizontal angle in degrees with similar meanings of subscript and postscript. Subscripts h, a and v denote horizontal, azimuth and vertical respectively. Thus B_h and B_v represents horizontal and vertical beamwidths of the antenna respectively, and λ_h and λ_v represent weighting factors for the horizontal and vertical beam pattern of the antenna in 3D antenna model [8], respectively. G_{max} and A_{max} denote the maximum antenna gain at the boresight of the antenna and maximum antenna attenuation at the sides and back of the boresight of the antenna respectively, in dB. For sake of simplicity we can neglect the the maximum attenuation factor A_{max} in (2). Without loss of generality we can assume maximum gain of 0 dB. Thus by putting $G_{max} = 0dB$ in (2), converting it from dB to linear form it can be simplified as:

$$G_k^n = 10^{-1.2\left(\lambda_v\left(\frac{\theta_k^n - \theta_{tilt}^n}{B_v}\right)^2 + \lambda_h\left(\frac{\phi_k^n - \phi_a^n}{B_h}\right)^2\right)} \quad (3)$$

For ease of expression we use following substitutions:

$$c_k^n = \frac{B_v^2 \lambda_h}{\lambda_v}\left(\frac{\phi_k^n - \phi_a^n}{B_h}\right)^2 ; h_k^n = \alpha \left(d_k^n\right)^{-\beta} ; \mu = \frac{-1.2\lambda_v}{B_v^2} \quad (4)$$

We assume that all the BS antennas transmit with same power. For such scenario, by using (3) in (1) and applying the substitutions in (**??**), the SIR in (1) can be written as:

$$\gamma_k^n = \frac{h_k^n 10^{\mu\left(\left(\theta_k^n - \theta_{tilt}^n\right)^2 + c_k^n\right)}}{\sum_{\forall m \in \mathcal{N} \setminus n}\left(h_k^m 10^{\mu\left(\left(\theta_k^m - \theta_{tilt}^m\right)^2 + c_k^m\right)}\right)} \quad (5)$$

Since our objective is to maximise hotspot user throughput without sacrificing non hotspot user throughput, so our problem statement becomes *optimize system wide antenna tilts to maximize the aggregate bandwidth normalised throughput for* *<u>all</u> users in the system*. Mathematically:

$$\max_{\boldsymbol{\theta}_{tilt}^{N}} \sum_{\forall s\in\mathcal{K}} \log_2\left(1+\gamma_k^n\left(\boldsymbol{\theta}_{tilt}^{N}\right)\right) \tag{6}$$

Note that γ_k^n is a function of vector of tilt angles of *all* sectors in WCS i.e. $\boldsymbol{\theta}_{tilt}^{N}$ where $N=|\mathcal{N}|$. Furthermore $\mathcal{K}$ can also be anticipated to be a large set. This indicates (6) is large scale nonlinear optimization problem. In next section we show how the paradigms of SO can be exploited to reduce to complexity of this solution to achieve a distributed near optimal solution.

III. DESIGNING A SO SOLUTION

In nature many systems can be observed to manifest perfect self organization. A detailed discussion on designing self organisation can be found in our work in [1] as well as in [12]. Here it would suffice to say that, for perfectly self organising solution, perfect objective may not be aimed for at system-wide level [12]. Rather,an *approximation of the objective* can be aimed for, given that it can be *decomposed* into local subproblems that can be solved at local level by the local entities of system. This phenomenon, in turn can approximately achieve the original system wide objective resulting into emergence of self organising behaviour [1], [12].

This design principle of self organisation when applied to our problem in (6) means, given the complexity of this problem, we need to 1)find an *approximation* that can be then *2)decomposed* down into locally solvable independent problems. And then we need to 3)determine the solution of that local problem. In following subsection we follow these three steps to achieve a novel self organising solution.

A. Approximating the Complex Problem with Simpler Problem

In order to simplify the problem in (6) such that it can be decomposed into local subproblems, we propose a novel concept of CG to effectively represent user geographical distribution in this context of tilt optimization. i.e. We propose to use hotspot's central locations as CG's of user distribution in a sector with respect which the antenna tilts of the system can be optimised. Since in case of hotspots based demography majority of user are in close vicinity of that central location of hotspot, so such optimization is bound to improve the overall system performance compared to arbitrary tilting, or the tilting does not aim at any particular point e.g. the one done for interference reduction [3] and [5] or no tilting. (This point will be further clarified in numerical results). The locations of of pop up hotspots can be easily determined by number of mechanisms including CCTV's surveillance cameras, through GPS or plethora of alternative location estimation algorithms, to be supplied to WCS as CG in an online manner.

If $\mathcal{S}$ denotes the set of all CG's in the system i.e. set of points representing geographical center of hotspots as shown in figure 2 with small circles, and we assume that each sector has one hotspot, our problem in (6) can be approximated as: *optimize system wide antenna tilts to maximize the aggregate bandwidth normalised throughput at CG's in whole system*.i.e.

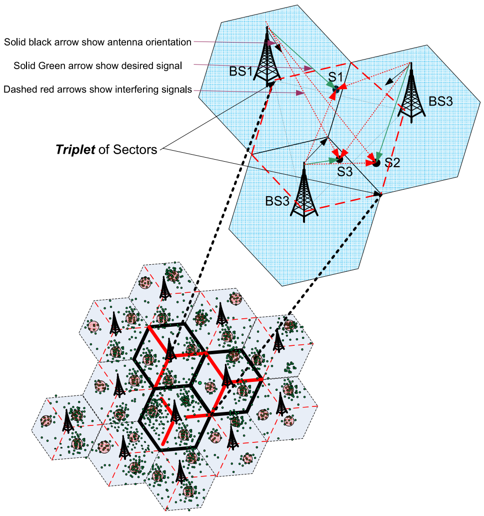

Fig. 2. A pictorial illustration of how hotspot based user distribution in WCS can be represented by single points called center of Gravity (shown by red circles)i.e. Figure also shows how system-wide optimization of tilts can be decomposed into optimization of tilts within each triplet independently.

$$\max_{\boldsymbol{\theta}_{tilt}^{N}} \sum_{\forall s\in\mathcal{S}} \log_2\left(1+\gamma_s^n\left(\boldsymbol{\theta}_{tilt}^{N}\right)\right) \tag{7}$$

B. Decomposing System-wide Problem into Local Subproblem

As discussed above, decomposition of (7) is necessary for a self organising solution [1], [12]. To enable this decomposition or localisation, we propose the concept of *triplet* as illustrated in figure 2. The reason to exploit triplet as a local monolithic entity for self organisation, is that it is *fixed* cluster of three adjacent and hence mutually *most interfering* sectors as shown in figure 2. Thus, firstly it will not require system-wide co-operations as suggested by some works, thereby avoiding the need for heavy signalling; and secondly, yet it incorporates the dominant interferer cells in the optimization process, thirdly it converts the large scale optimization problem in (7) to a small scale optimization problem that is solvable with state of the art optimization techniques as we show in the following. Let $\theta_{tilt}^{T_i}$ denote vector of tilt angle of sectors within i^{th} triplet, now the local optimization problem to be solved and executed within a triplet is given as:

$$\max_{\boldsymbol{\theta}_{tilt}^{T_i}} \sum_{\forall s\in\mathcal{S}_i} \log_2\left(1+\hat{\gamma}_s^n\left(\boldsymbol{\theta}_{tilt}^{T_i}\right)\right) \tag{8}$$

Whereas $\hat{\gamma}$ shows that SIR here is an approximate SIR as it considers interference from the two most interfering adjacent sectors only. It can be written as:

$$\hat{\gamma}_p^n\left(\boldsymbol{\theta}_{tilt}^{T}\right)=\frac{h_p^n 10^{\mu\left(\left(\theta_p^n-\theta_{tilt}^n\right)^2+c_p^n\right)}}{\sum_{\forall t\in\mathcal{T}\backslash n}\left(h_p^t 10^{\mu\left(\left(\theta_p^t-\theta_{tilt}^t\right)^2+c_p^t\right)}\right)} \tag{9}$$

The pros and cons of this localisation of problem in (7) are highlighted in III-D. In next section we present methodology to solve this local problem.

C. Solving Local Problem

If C is the total achievable bandwidth normalised throughput in a given triplet at the three CG's within (subscript dropped for simplicity of expression) then:

$$C = \log_2\left(1 + \hat{\gamma}_1^1\right) + \log_2\left(1 + \hat{\gamma}_2^2\right) + \log_2\left(1 + \hat{\gamma}_3^3\right) \quad (10)$$

where postscript denote sector number and subscripts denote CG within a triplet, as shown in figure 2. Since number of optimization parameters is only three now and their range is also finite i.e.$0 < \theta < 90$, so the solution of (8) can be easily determined using a non linear optimization technique that can tackle a small scale non convex optimization problem.[2] Observing that (10) is twice differentiable, we solve this sub problem using sequential quadratic programming (SQP). For sake of clarity, we drop the subscript tilt. Instead we use subscript to present the association with sector in the triplet. Then the problem in (8) can be written in the standard form:

$$\min_{\boldsymbol{\theta}} \quad -C\left(\boldsymbol{\theta}\right) \quad (11)$$

subject to: $g_j\left(\theta_j\ \right) \ < \ 0 \quad ,j \ = \ 1, 2, 3$
where $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3]$ and $g_j(\theta_j) = \theta_j - \frac{\pi}{2}$. The Lagrangian of constrained optimization problem in (11) can be written as:

$$\mathcal{L}\left(\boldsymbol{\theta}, \boldsymbol{\lambda}\right) = C\left(\boldsymbol{\theta}\right) - \boldsymbol{\lambda}^T \boldsymbol{g} \quad (12)$$

$$\mathcal{L}\left(\boldsymbol{\theta}, \boldsymbol{\lambda}\right) = C\left(\boldsymbol{\theta}\right) - \sum_{j=1}^{3} \lambda_j (\theta_j - \frac{\pi}{2}) \quad (13)$$

If $\hat{\boldsymbol{H}}$ denotes the approximate of the Hessian matrix $\boldsymbol{H}$, then we can define quadratic subproblem to be solved at r^{th} iteration of SQP as follows:

$$\min_{\mathbf{w} \in R^J} \quad \frac{1}{2}\mathbf{w}^T \hat{\mathbf{H}} \left(\mathcal{L}\left(\boldsymbol{\theta}, \boldsymbol{\lambda}\right)\ \right)_r \mathbf{w} + \bigtriangledown C(\boldsymbol{\theta})_r \mathbf{w} \quad (14)$$

subject to: $w_j + \theta_{j_r} - \frac{\pi}{2} < 0 \quad j = 1, 2, 3$

At each iteration the value of $\hat{\mathbf{H}}$ can be updated using the Broyden-Fletcher -Goldfarb -Shanno (BFGS) approximation method. Once the Hessian is known the problem in (14) is a quadratic programming problem that can be solved using standard methods e.g. gradient projection method in [13].

Through the above steps of SQP, the problem in (8) can be solved within each triplet independently to determine the optimal tilt angle to be adapted and maintained by each triplet for given locations of hotspots within that triplet. The execution of these solutions in each triplet in the whole WCS independently, results in achievement of the system wide objective in (6), *approximately*. We call this framework SOT-HR i.e. SO of Tilts for Hotspot Relief

TABLE I

SYSTEM LEVEL SIMULATION PARAMETERS

Parameters	**Values**
System topology	19 BS×3 sector, Frequency Reuse 1
BS Transmission Power	39 dBm
Cell Radius,BS and user height	600, 32 and 1.5 meters respectively
User antenna gain	0 dB (Omini directional)
B_h,B_v	70^0,10^0
$\lambda_v = \lambda_h$	0.5
G_{max},A_{max}	18, 20 dB
Frequency	2 GHz
Pathloss model	3GPP Urban Macro
Bandwidth	5 MHz
Shadowing standard deviation	8 dB
Scheduling	Round Robin
Total user population	10000 users
Hot Spots Radius	10% of cell radius (located randomly)
% of users in hot spots	50%

D. Implications of SOT-HR

It can be seen that the subproblem in (8) does not aim to optimise system wide antenna tilts. This feature has one major advantage i.e. this subproblem can be solved independently in within each triplet without requiring coordination with rest of the antennas in other triplets. The cost of this advantage is that the system-wide globally optimal performance is neither aimed for nor achieved through SOT-HR; however, it is just like the case that in nature SO systems do aim for perfectly optimal objectives. For instance, common cranes never fly in perfect V-shape, but even maintaining a near V-shape increases their group flight efficiency by 70% [14]. Furthermore, as postulated in [12], one of the four main paradigms for designing SO into system is that, for perfect SO perfect objectives need not be aimed for. So here the SO nature of the proposed solution is perfect but at cost of *sub-optimal global objective*.

IV. PERFORMANCE EVALUATION

In order to demonstrate the potential of SOT-HR we evaluate its performance for a stand alone triplet (figure 3) as well at system level (figure 4) by implementing it in a system level simulator that models an OFDMA based generic WCS. Table I shows key simulation parameters used.

Figure 3 plots $C/3$ in (10) for a stand alone triplet for two different set of locations of CG i.e. hotspot centres. Two key observations can be made from this result. Firstly, SOT-HR can improve spectral efficiency of users at hotspots by upto 1b/s/Hz (2 to 3 and 3 to 4 b/s/Hz in bottom and top left of figure 3, receptively). Secondly, the optimal value of tilt angles are strongly dependent on the location of hotspots. This observation highlights the need for SO of tilts to cope with spatio temporally dynamic pop hotspots; and SOT-HR actually enable this SO of tilts in a pragmatic way. This also is demonstrated by system level performance of SOT-HR in figure 4.

For a comparative analysis of SOT-HR performance is also evaluated for four other fixed tilting options with arbitrary tilt values of $0^0, 5^0, 20^0$ or 25^0 . Unlike these tilting options where all sectors in system have fixed tilts, SOT-HR optimally sets value of each sector independently in each triplet based on location of hotspots. A significant gain in throughput achievable

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[2] Non convexity of C can be observed by plotting C against θ. See in figure 3

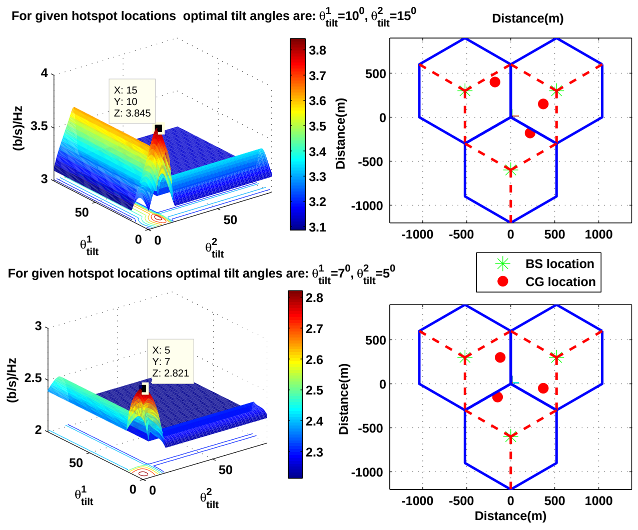

Fig. 3. Bandwidth normalised average throughput per link $\frac{C}{3}$ plotted for a stand alone triplet against tilts of two sectors while third is fixed at 0^0 degree.

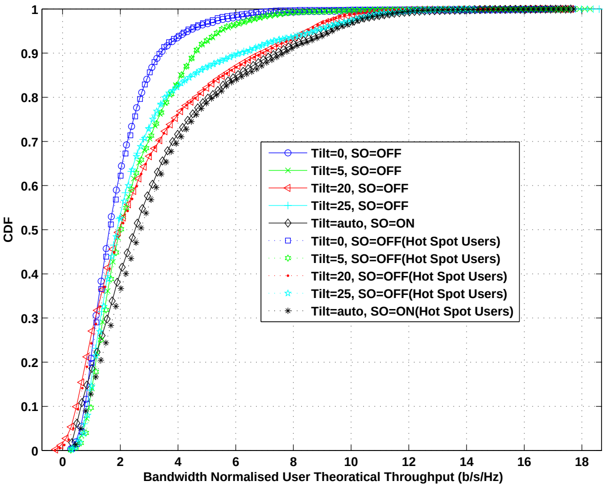

Fig. 4. Throughput for all users and users in the hotspot. Results for fixed tilts are denoted by legend '*Tilt=x,SO=OFF*'. Where x denotes θ_{tilt} . The results with SOT-HR in place are denoted with legend '*Tilt=auto,SO=ON*'

with SOT-HR can be observed in figure 4 compared to other fixed tilt configurations. Another important observation is that for all other fixed tilt configurations throughput for hotspot only and all users are almost same. But SOT-HR provides a relatively higher throughput to hotspot users. This is because the SOT-HR optimises the antennas with respect to hotspot centers i.e. C.Gs in the sectors. CDF's also show that SOT-HR does not degrade the overall average user throughput.

V. CONCLUSION AND FUTURE WORK

A novel framework SOT-HR for spectral efficiency optimization in presence of hotspots through self organisation of antenna tilts in distributed and dynamic manner is presented. Both numerical and system level simulations results presented show that a significant gain in spectral efficiency of up to 1b/s/Hz achievable with the proposed framework. From practical implementation point of view, the main advantage of SOT-HR is that it requires a negligible amount of only local signalling i.e. among the sectors within triplet to determine the location of hotspot. This signaling can be done through X2 interface and most importantly it can be done only when location of whole hotspot changes i.e. tracking of individual user location is not required. In future this framework would be extended for interference avoidance at Femto cells instead of hotspots.

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REFERENCES

[1] A. Glenn, A. Imran, M. Imran, and B. Evans, "A suvey of self organization in wireless cellular systems," *submitted to IEEE Journal of Surveys and Tutorials*, 2011.

[2] V. Wille, M. Toril, and R. Barco, "Impact of antenna downtilting on network performance in GERAN systems," *Communications Letters, IEEE*, vol. 9, no. 7, pp. 598 – 600, july 2005.

[3] J. Wu, J. Chung, and C. Wen, "Hot-spot traffic relief with a tilted antenna in CDMA cellular networks," *in IEEE Transactions on Vehicular Technology*, vol. 47, no. 1, pp. 1 –9, Feb 1998.

[4] I. Siomina, P. Varbrand, and D. Yuan, "Automated optimization of service coverage and base station antenna configuration in UTMS networks," *Wireless Communications, IEEE*, vol. 13, no. 6, pp. 16 –25, Dec. 2006.

[5] R. Abou-Jaoude, N. Ulhaq, and C. Hartmann, "HSDPA throughput optimization with antenna tilt and pilot power in a moving hot-spot scenario," in *IEEE 70th Vehicular Technology Conference,2009-Fall(VTC'09-Fall)*, 2009, pp. 1 –5.

[6] M. N. ul Islam, R. Abou-Jaoude, C. Hartmann, and A. Mitschele-Thiel, "Self-optimization of antenna tilt and pilot power for dedicated channels," in *Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt), 2010 Proceedings of the 8th International Symposium on*, May 2010, pp. 196 –203.

[7] N. Zheng, P. Michaelsen, J. Steiner, C. Rosa, and J. Wigard, "Antenna tilt and interaction with open loop power control in homogeneous uplink LTE networks," in *IEEE International Symposium on Wireless Communication Systems,2008,(ISWCS '08)*, Oct. 2008, pp. 693 –697.

[8] I. Viering, M. Dottling, and A. Lobinger, "A mathematical perspective of self-optimizing wireless networks," *IEEE International Conference on Communications, 2009,(ICC '09)*, pp. 1 –6, June 2009.

[9] O. Yilmaz, S. Hamalainen, and J. Hamalainen, "Comparison of remote electrical and mechanical antenna downtilt performance for 3GPP LTE," in *Proc. IEEE 70th Vehicular Technology Conf. 2009-Fall (VTC'09-Fall)*, 2009, pp. 1–5.

[10] R. Razavi, S. Klein, and H. Claussen, "Self-optimization of capacity and coverage in LTE networks using a fuzzy reinforcement learning approach," in *Proc. IEEE 21st Int Personal Indoor and Mobile Radio Communications Symp,2010,(PIMRC'10)*, 2010, pp. 1865–1870.

[11] X. Lu, E. Kunnari, J. Leinonen, O. Piirainen, M. Vainikka, and M. Juntti, "LTE uplink power control and base station antenna down tilt in a 3D channel model," in *Proc. European Wireless Conf. (EW)*, 2010, pp. 377–381.

[12] C. Prehofer and C. Bettstetter, "Self-organization in communication networks: principles and design paradigms," *Communications Magazine, IEEE*, vol. 43, no. 7, pp. 78 – 85, july 2005.

[13] P. Gill, W. Murray, and M. H. Wright, *Practical Optimization.* London, Academic Press, 1981.

[14] P. B. S. Lissaman and C. A. Shollenberger, "Formation flight of birds," *Science*, vol. 168, no. 3934, pp. 1003–1005, 1970. [Online]. Available: http://www.sciencemag.org/cgi/content/abstract/168/3934/1003