Effect of Inaccurate Position Estimation on Self-Organising Coverage Estimation in Cellular Networks

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Abstract—Requirement for low operating and deployment costs of cellular networks motivate the need for self-organisation in cellular networks. To reduce operational costs, self-organising networks are fast becoming a necessity. One key issue in this context is self-organised coverage estimation that is done based on the signal strength measurement and reported position information of system users. In this paper, the effect of inaccurate position estimation on self-organised coverage estimation is investigated. We derive the signal reliability expression (i.e. probability of the received signal being above a certain threshold) and the cell coverage expressions that take the error in position estimation into consideration. This is done for both the shadowing and non-shadowing channel models. The accuracy of the modified reliability and cell coverage probability expressions are also numerically verified for both cases.

I. INTRODUCTION

Every day more and more people are using their mobile devices to access the internet and the demand for wireless access is increasing exponentially which causes a rapid growth in mobile communications. Consequently, this rapid growth is causing the distribution and maintenance of cellular networks to become more and more complex, expensive and time consuming. Hence, there is an urgent need for a new functionality in cellular networks which would cope with this increased complexity, while reducing cost and maintenance time.

Self-organisation is an adaptive functionality where the network can detect changes and based on these detected changes, makes intelligent decisions to maximize or minimize the effects of the changes [1]. Self-organisation is effectively the only feasible way of achieving optimal performance in future wireless cellular networks in a cost effective manner [2]. Hence, standardization bodies for Long term evolution (LTE) and LTE-Advance have identified self-organisation as not just an optional feature but an inevitable necessity in the future wireless systems [3].

One of the ways of enabling self-organisation in cellular networks is by coverage estimation. The cell coverage is defined as the maximum distance that a mobile user can be from its service base station (BS) while still maintaining satisfactory service [4].

There are different ways of performing coverage estimation and each of them has their own advantage(s) and disadvantage(s), but none of them are 100% accurate [5]–[8]. Bernardin et al. [5] used a linear regression of radio frequency (RF) signal strength samples to accurately determine the effective cell radius. The accuracy of this estimation was quantified both as a radius uncertainty (e.g., ±100 meters) and as a coverage reliability error. They showed that if the estimate of the cell radius meets the desired accuracy then the corresponding estimates of coverage reliability (both area and edge) are more than sufficiently accurate. However, in real life, the actual shape of the cell is irregular due the random shadowing around the cell (e.g., building and etc.), so the coverage probability estimation will be a more ideal solution than radius estimation.

Mobile location estimation is becoming an important service for mobile networks. In heterogeneous networks with self-organising networks (SON) functionality, the user terminals (UTs) can report their location to the macro cell which then associate each UT to a small cell based on their reported position. It is well known that the global positioning system (GPS) can provide accurate location estimation, but it does not perform well in urban areas. This is because satellite signals are often reflected, deflected or blocked by high building which results in inaccurate estimation [6]. An alternative to satellite-based positioning is positioning based on cellular networks which also have similar limitations, especially the low accuracy (in the range of 100 m) [7]. Roos et al. [8] proposed a statistical propagation model that describes the distribution of received signal power at any given location and used the model for estimating the mobile units' location when the received power is observed.

In this paper, the effect of inaccurate position estimation on self-organising coverage estimation is investigated. We derive the coverage probability by taking into account the error in the GPS based location estimation and compare it with the simulation approach coverage probability, for various GPS approximation errors. Moreover, the reliability of the RF signal received by the UT, i.e. the probability of the received signal being above a threshold, is also investigated for both shadowing and non-shadowing cases. The rest of this paper is organised as follows: Section II, introduces the system model

and defines the relevant performance measures. Section III, formulates the reliability and coverage probability when the GPS error is incorporated. Section IV, numerically verifies the accuracy of the derivations. And finally, conclusions are made in Section V.

II. SYSTEM MODEL AND PERFORMANCE METRICS

In this paper, we utilize two fundamental measures of reliability of RF coverage, i.e., cell edge reliability and cell coverage probability, to demonstrate the effect of inaccurate position estimation in self-organizing cellular networks.

A. Propagation Model

The degradation of signal quality is usually assumed to be due to three different causes: fast fading, path loss and slow fading, also known as shadowing. In this work we focus on path loss and shadowing in our derivations. The signal propagation model we employ is as follows

$$P_r(p) = \left(\frac{p}{p_0}\right)^{-\eta} \frac{P_t}{Pl(p_0)} \Phi, \qquad (1)$$

where P_r, P_t, p, and η denote receive and transmit power, propagation distance, and path loss exponent, respectively. The parameter p_0 denotes to the reference distance with a known path loss, $Pl(p_0)$. The shadowing effect is modelled by the random variable, Φ, which follows a log-normal distribution such that $10 \log_{10} \Phi$ follows a zero mean Gaussian distribution with standard deviation σ in dB.

B. Cell Edge Reliability

In cellular networks, a minimum signal strength P_{min} is usually required to maintain the desired quality of service. We define the cell edge reliability as the probability that the received power strength measured on a circular contour at the cell edge will exceed or meet a desired quality threshold. In addition, the reliability metric can also be defined for any point within the cell coverage, i.e. $Pr[P_r(p) \geq P_{min}]$, $\forall 0 \leq p \leq R$, where R is the radius of the cell.

C. Cell Coverage Probability

We define the cell coverage probability, $\mathcal{C}$, as the fraction of cell area where the received power is above the minimal required signal strength P_{min}. The cell coverage probability is obtained by integrating the contour probability over the entire coverage area of the cell, i.e. across all contours of the cell including the cell edge, and dividing it by the cell area. Hence, we can express the cell coverage probability as

$$\mathcal{C} = \frac{1}{A} \int_A p.\mathbb{P}\left[P_r(p, \phi) \geq P_{min}\right] \mathrm{d}p \, \mathrm{d}\phi , \qquad (2)$$

where A denotes the cell area, as illustrated in Fig. 1. In this paper we consider a single cell deployment.

D. GPS Error Modelling

We consider that a central controller utilizes GPS to estimate the position of each UT. The GPS system has an uncertainty region of radius, r, which implies that given the UT reported coordinates are (c, d), the actual UT position is a point with coordinate (x, y) that satisfies

$$(x - c)^2 + (y - d)^2 \leq r^2. \qquad (3)$$

Considering a heterogeneous network where the macro cells provide ubiquitous coverage while the small cells are for high data rate transmission. Furthermore, the macro cells are aware of the exact position of the small cells and they allocate UTs to each small cell based on the GPS report of the UTs position. As a result of the uncertainty in GPS estimation, some of the UTs might not be in the coverage of their allocated small cell. In this work, we aim to estimate the impact of the position estimation error on the cell edge reliability and cell coverage probability.

III. COVERAGE RELIABILITY ESTIMATION

In this section, we derive the modified expression of the cell edge reliability and cell coverage probability that take the GPS estimation error into consideration. We first formulate the expressions for the simplified case without shadowing, which we later extend to the shadowing case.

A. Case without Shadowing

In the case without shadowing and GPS uncertainty, the cell edge reliability and cell coverage probability are equivalent and can be expressed as

$$Pr[P_r(p) \geq P_{min}] \equiv \mathcal{C} = \begin{cases} 1 & p \leq p_0 \left(\frac{P_{min} Pl(p_0)}{P_t}\right)^{\eta} \\ 0 & p > p_0 \left(\frac{P_{min} Pl(p_0)}{P_t}\right)^{\eta} \end{cases} . \qquad (4)$$

When the GPS uncertainty is considered, for us to estimate the cell edge reliability, we need to estimate the area of all possible UT positions, i.e., (3) that lie within the coverage the cell. Given the cell centre coordinates, (a, b), cell coverage radius, $R = p_0 \left(\frac{P_{min} Pl(p_0)}{P_t}\right)^{\eta}$, and the reported UT coordinates, (c, d), we are interested in finding the portion (area) of the dotted circle that lies within the cell coverage, as illustrated in Fig 1. By using the laws of trigonometry we obtain this area as

$$\bar{A} = \begin{cases} \pi r^2 & 0 < p \leq R - r \\ \pi r^2 - \left[\frac{(\beta - \sin \beta) r^2}{2} - \frac{(\theta - \sin \theta) R^2}{2}\right] & R - r < p \leq \sqrt{R^2 - r^2} \\ \left[\frac{(\beta - \sin \beta) r^2}{2}\right] + \left[\frac{(\theta - \sin \theta) R^2}{2}\right] & \sqrt{R^2 - r^2} < p < R \end{cases} , \qquad (5)$$
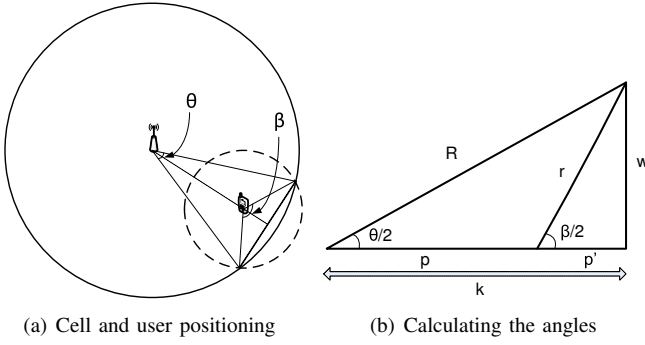
(a) Cell and user positioning   (b) Calculating the angles

Fig. 1.   *The way the cell and the user are positioned and the angles created. (b) shows the triangle created in (a).*

where θ and β are obtained as

$$\theta = 2.\cos^{-1}\left[\frac{R^2+p^2-r^2}{2.p.R}\right] \quad \text{and}$$

$$\beta = 2.\cos^{-1}\left[\frac{R^2-p^2-r^2}{2.p.r}\right], \qquad (6)$$

respectively. The parameter $p = \sqrt{(c-a)^2+(d-b)^2}$ is the distance between the reported UT position and the cell centre, as illustrated in Fig. 1.

Consequently, the reliability of the received signal at any point in the interval $0 \leq p \leq R$ can be expressed as

$$Pr_{ns}[P_r(p) \geq P_{min}] = \frac{\bar{A}}{\pi r^2} \equiv$$

$$\begin{cases} 1 & 0<p\leq R-r \\ 1-\left[\frac{(\beta-\sin\beta)}{2\pi}-\frac{(\theta-\sin\theta)}{2\pi}\left(\frac{R}{r}\right)^2\right] & R-r<p\leq\sqrt{R^2-r^2}, \quad (7) \\ \frac{(\beta-\sin\beta)}{2\pi}+\frac{(\theta-\sin\theta)}{2\pi}\left(\frac{R}{r}\right)^2 & \sqrt{R^2-r^2}< p <R \end{cases}$$

for the case without shadowing. This clearly shows that 100% reliability is only obtained when the reported UT position is at a distance such that $0 < p \leq R-r$. We can therefore obtain the cell area coverage probability according to (2) as

$$C_{ns} = \frac{R^2-(R-r)^2}{R^2} + \frac{1}{\pi R^2}\left(\int_{R-r}^{\sqrt{R^2-r^2}} p(1-\bar{A}_1)\mathrm{d}p \right. $$
$$\left. + \int_{\sqrt{R^2-r^2}}^{R} p(\bar{A}_1+2\bar{A}_2)\mathrm{d}p\right) \qquad (8)$$

for the case without shadowing, where $\bar{A}_1 = \frac{(\beta-\sin\beta)}{2\pi} - \frac{(\theta-\sin\theta)}{2\pi}\left(\frac{R}{r}\right)^2$ and $\bar{A}_2 = \frac{\theta-\sin\theta}{2\pi}\left(\frac{R}{r}\right)^2$, with θ and β defined in (6). Note that the modified reliability and coverage probability expressions in (7) and (8), respectively, reverts to the expression in (4) when the GPS error radius $r = 0$.

B. Case with Shadowing

The cell edge reliability and the coverage probability for the shadowing case without GPS error are given in [9], [10] as

$$Pr[P_r(p) \geq P_{min}] = \frac{1}{2} - \frac{1}{2}\text{erf}\left(a + b\ln\frac{p}{R}\right) \quad \text{and} \qquad (9)$$

$$C = \frac{1}{2} - \frac{1}{R^2}\int_0^R p\,\text{erf}\left(a+b\ln\frac{p}{R}\right)\mathrm{d}p, \qquad (10)$$

where $a = \frac{\left(P_{min}(\text{dBm})-P_t(\text{dBm})+Pl(p_0)(\text{dB})+10\eta\log_{10}\frac{R}{p_0}\right)}{\sigma\sqrt{2}}$, and $b = (10\eta\log_{10}e)/\sigma\sqrt{2}$.

Given the reliability expression in (9), we can obtain the reliability expression at position p, for the case with GPS uncertainty by first defining the probability $Pr[P_r(\bar{p}) \geq P_{min}]$, where

$$\bar{p} = \sqrt{p^2+\kappa^2-2p\kappa\cos\phi} \qquad (11)$$

such that $0 \leq \kappa \leq r$ and $0 \leq \phi \leq 2\pi$ defines every possible point as a result of the GPS error. Consequently, the reliability at position p due to the GPS uncertainty, can be expressed as

$$Pr_s[P_r(p) \geq P_{min}] = \qquad (12)$$
$$\frac{1}{2\pi r}\int_0^r\int_0^{2\pi}\left[\frac{1}{2}-\frac{1}{2}\text{erf}\left(a+\frac{b}{2}\ln\frac{p^2+\kappa^2-2p\kappa\cos\phi}{R^2}\right)\right]\mathrm{d}\phi\mathrm{d}\kappa,$$

when shadowing is considered.

Furthermore, the cell coverage probability can be expressed according to (2) as

$$C_s = \frac{1}{\pi r R^2}\int_0^R\int_0^r\int_0^{2\pi} p\left[\frac{1}{2}-\frac{1}{2}\text{erf}\left(a \right.\right. $$
$$\left.\left. + \frac{b}{2}\ln\frac{p^2+\kappa^2-2p\kappa\cos\phi}{R^2}\right)\right]\mathrm{d}\phi\mathrm{d}\kappa\mathrm{d}p, \qquad (13)$$

when shadowing is considered.

The modified reliability expression and coverage probability derived in (12) and (13), respectively, for the case with GPS error and shadowing reverts back the expression (9) and (10), when the GPS error radius, $r = 0$, since κ and ϕ will also be zero.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section we verify the accuracy of our modified reliability and the cell coverage probability expressions that were derived for both the shadowing and non-shadowing cases numerically. We consider the scenario with a single base station transmitting at fixed power $P_t = 46$dBm. We assume that minimum received signal that the UT can effectively decode, $P_{min} = -84.5$dBm. Hence, we obtain the cell coverage radius, R from (1) with $\Phi = 1$ and using the parameter in Table I.

We randomly position $100,000$ UTs within the coverage of the cell (i.e., circular cell with radius R), to represent the reported position. We incorporate the GPS error to the reported position of each UT by adding a random displacement, $(\bar{r}_i, \delta,$ for each UT, such that $0 \leq \bar{r}_i \leq r$, and $0 \leq \delta \leq 2\pi$ to obtain the exact UT position .

For the case without shadowing, in order to verify the coverage probability C_{ns} in (8), we simply evaluate the received

Parameters	Values
Standard Deviation (σ)	7
Path Loss Exponent (η)	3.5
Reference Distance (p_0)	1 m
Path Loss at p_0 ($Pl(p_0)$)	34.5 dB
Power Transmitted (P_t)	46 dBm
Threshold (P_{min})	-84.5 dBm
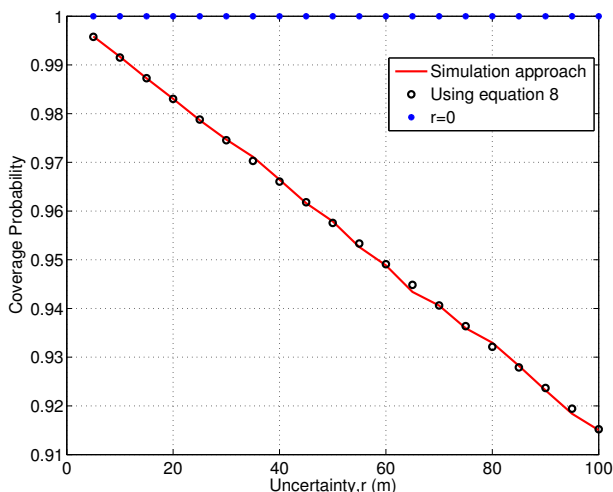
Fig. 2. The coverage probability as a function of GPS error radius when shadowing is not considered.

Fig. 3. The coverage probability of the users under shadowing for different uncertainties (r).

signal strength at exact UT position, i.e. P_r, using (1) with $\Phi = 1$. Thereafter, we obtain the percentage of UT with $Pr \geq P_{min}$, which we refer to as the simulation approach coverage probability.

In Fig.2, we compare the exact coverage probability obtained from (8) and the coverage probability using the simulation approach for GPS approximation error radius, r, ranging from 5 to 100, when shadowing is not considered. As it can be seen, our GPS error modified coverage probability expression tightly matches with the simulation approach. Furthermore, the coverage probability reduces linearly with the GPS error radius. At $r = 20$m, approximately 2% of the UT will be out of coverage, which further degrades to about 6% at $r = 70$m. Hence, the GPS error results into coverage gaps in the network. We can also observe that as GPS radius $r \to 0$, the coverage probability $\mathcal{C}_{ns} \to 1$, which matches the insight drawn in our analysis.

For the case with GPS error and shadowing, we randomly position 10,000,000 UTs within the coverage of the cell, to represent the reported position. Similar to the non-shadowing case, we obtain the simulation approach coverage probability by finding the percentage of UTs with $P_r \geq P_{min}$, where P_r calculated from (1) and $10 \log_{10} \Phi$ follows a zero mean Gaussian distribution with standard deviation σ. The results
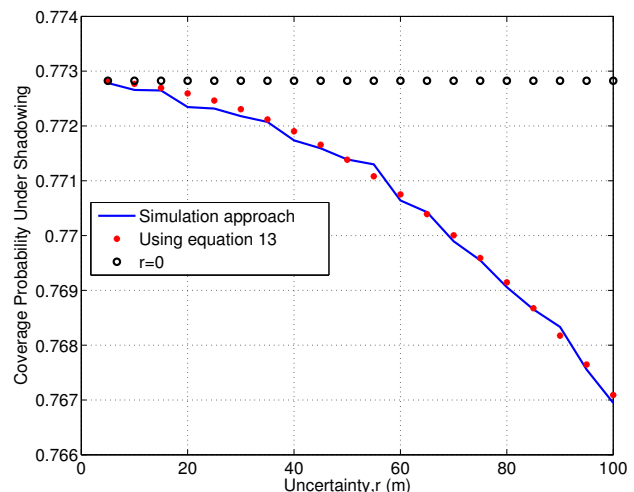
in Fig.3 shows that, when shadowing is considered, our GPS error modified coverage probability in (13) matches tightly with the simulation approach. Similar to the non-shadowing case, the coverage probability reduces with the GPS error radius. Furthermore, the GPS error also results into coverage gaps in the network, since the coverage probability with GPS error, i.e., $r > 0$ is lower than that without GPS error, i.e., $r = 0$. It can also be seen that as the GPS error radius $r \to 0$, the coverage probability $\mathcal{C}_s \to \mathcal{C}$ in (10), which is inline with the insight drawn earlier.

In Fig.4, we plot the performance degradation (PD) as result of the GPS error, for shadowing standard deviation $\sigma = 7, 9$ and 12dB. We define the performance degradation as $PD = \frac{\mathcal{C}_s - \mathcal{C}}{\mathcal{C}}$, where $\mathcal{C}$ defined in (10) is the cell coverage probability for the shadowing case without GPS error. It can be observed in Fig.4 that the performance becomes more degraded as the shadowing standard deviation σ reduces. This implies that the GPS approximation error is less severe on the coverage as σ increases. The reason for this is that increasing σ introduces more randomness to the received signal; hence uncertainty/randomness created by the GPS error would have more impact on a lower σ.

In Fig.5, we plot the reliability of the RF signal received by the UT positioned at a distance p from the cell centre, i.e. $Pr[P_r(p) > P_{min}]$ such that $0 \leq p \leq R$, $P_{min} = -84.5$dBm, $\sigma = 7$dB and $r = 50$, for both the shadowing and non-shadowing cases. It can be seen that for the non-shadowing case with GPS error, the reliability, $Pr_{ns}[P_r(p) > P_{min}] = 1$ when $p \leq R - r$, and depreciates from this value when $p > R - r$. Whereas, $Pr[Pr(p) > P_{min}] = 1, \forall 0 \leq p \leq R$, when there is no GPS error and shadowing. For the case with shadowing, it can be seen in Fig.5 that the reliability $Pr_s[Pr(p)]$ obtained when there is GPS error is always less than the reliability without GPS error.
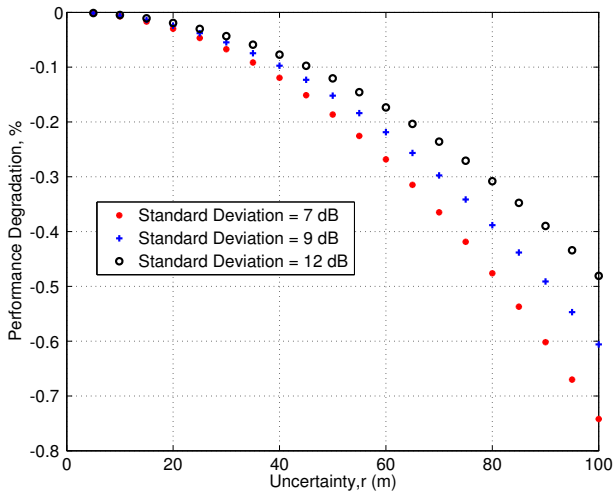
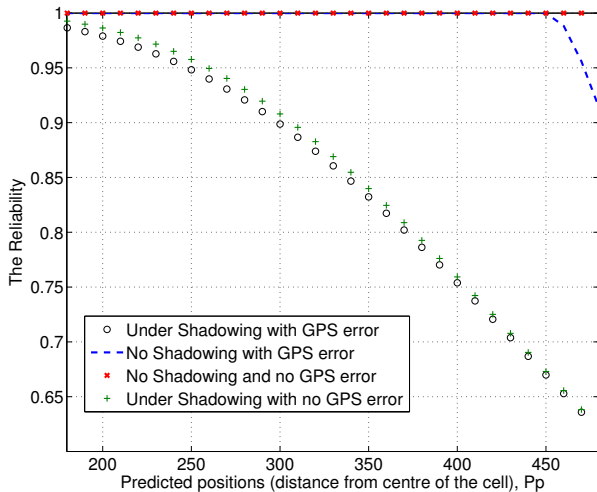Fig. 4.   Coverage degradation as a result of GPS error

Fig. 5.   Reliability of UT at distance p from the cell centre, for the shadowing and non-shadowing cases.

V. CONCLUSIONS

In this paper, we have investigated the effects of inaccurate position estimation on self-organising networks. We derived the reliability and cell coverage expressions that take the error in position estimation into consideration, for both the shadowing and non-shadowing cases. The accuracy of the modified reliability and cell coverage probability expressions were numerically verified for both cases. The proposed GPS error modified coverage probability expression tightly matched the simulation approach. Furthermore, the coverage probability decreased with the GPS error radius in both the shadowing and non-shadowing cases. The reliability of the shadowing case was shown to be approximately equal to 1 when the UT is positioned close to the cell centre, with a steady depreciation as the UT moves away from the cell centre.

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